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ON A PROGRAMME FOR THE BALANCING CALCULATION OF FLEXIBLE ROTORS WITH THE INFLUENCE COEFFICIENT METHOD

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ABSTRACT. This paper presents the influence coefficient method of determining the locations of unbalances on a flexible rotor system and the correction weights. A computer software for calculating the at-the-site balancing of a flexible rotor system was created using C^{++} language at the Hanoi University of Technology. This software can be used by balancing flexible rotors in Vietnam.

1. Introduction

The well-known methods of the at-the-site balancing of flexible rotors (the method of three time starting the trial weights, the vector triangle, the sensitivity) were successfully used to balance separate flexible rotors at the site. However, the efficiency of these balancing methods depends a lot on the correctness of the analysis of the vibration modes of separate rotors. Nowadays, rotors are manufactured longer and longer, many rotors are connected with each other. After manufacture, rotors are separately balanced before leaving the production workshop, but by connecting many rotors together, the separate balance status disappears due to mutual interaction of the residual unbalance remaining in each rotor which causes changes in the vibration of the entire system. The methods of separate rotor balancing may reduce vibration of the balanced rotor, but may increase vibration in many points in the other rotors of the system. In order to work safely, the vibration rate in all points of the rotor system, in all regimes, must lie within the permitted standards. Therefore the entire system of rotors must be balanced.

In this paper, the author present the influence coefficient method for balancing flexible rotors [1, 2, 3]. This method is dependent on the basic principle that the influence coefficient matrix is square. In actual balancing, however, the influence coefficient matrix is not necessarily square but is often a non-square matrix. The least-squares balancing method is a method in which correction weights are calculated under the condition of minimizing the sum of the squares of residual vibrations. From this method the computer software for the calculation of the at-the-site balancing of a flexible rotors system was created using C^{++} language at the Hanoi University of Technology.

2. Theoretical basis of a programme for balancing calculation

2.1. Concept of influence coefficient

Let us call $\overline{r}_j$ the vibration at the measured point j ($j = 1, \ldots, J$, depending on the measured point and the speed number), $\overline{r}_{jk}$ measurement results at j due to unbalance $\overline{U}$ in plane k at rotor speed Ω, we obtain the following formula:

$$\overline{r}_{jk} = \overline{\alpha}_{jk} \cdot \overline{U}_k, \tag{2.1}$$

where $\overline{\alpha}_{jk}$ is the proportion coefficient. This coefficient shows the influence of unbalance $\overline{U}_k$ on the measurement results at j^{th} measured point and is called the influence coefficient.

For convenience, let's have $\overline{r}_{jk}$ and $\overline{U}_k$ in the form of complex numbers, therefore $\overline{\alpha}_{jk}$ will also be calculated in complex number.

2.2. Determination of influence coefficients with measurement of vibration

The initial unbalance vibration at the measured point j, ($j = 1, \ldots, J$) is $\overline{r}_j^A$ vibration at j^{th} measured point with trial weight $\overline{U}_k$ is $\overline{r}_{jk}^M$ and we have

$$\overline{r}_{jk} = \overline{r}_{jk}^M - \overline{r}_j^A \tag{2.2}$$

From (2.1) we will have

$$\overline{\alpha}_{jk} = \frac{\overline{r}_{jk}}{\overline{U}_k} = \frac{\overline{r}_{jk}^M - \overline{r}_j^A}{\overline{U}_k} \tag{2.3}$$

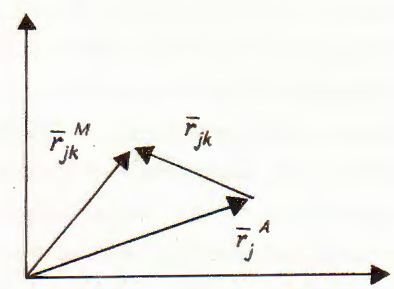

Fig. 1

The unit of $\overline{\alpha}_{jk}$ is $[m/kg]$ or $[mm/g]$. By changing the test weights at the balancing plane k ($k = 1, \ldots, K$) we will determine the influence coefficients $\overline{\alpha}_{jk}$ ($j = 1, \ldots, J$), ($k = 1, \ldots, K$).

2.3. Influence coefficient matrix and determination of the correction weights

The vibration at j^{th} point on the rotor due to separate unbalancing $\overline{U}_k$ ($k = 1, \ldots, K$) at all balancing planes according to formula (2.1) is

$$\bar{r}_j = \sum_{k=1}^{K} \bar{r}_{jk} = \sum_{k=1}^{K} \overline{\alpha}_{jk}\overline{U}_k \quad (j = 1, \ldots, J). \tag{2.4}$$

The system of algebraic equation (2.4) may be rewritten in the matrix form as follows

$$\begin{bmatrix} \bar{r}_1 \\ \bar{r}_2 \\ \vdots \\ \bar{r}_J \end{bmatrix} = \begin{bmatrix} \overline{\alpha}_{11} & \overline{\alpha}_{12} & \ldots & \overline{\alpha}_{1K} \\ \overline{\alpha}_{21} & \overline{\alpha}_{33} & \ldots & \overline{\alpha}_{2K} \\ \vdots & \vdots & \ddots & \vdots \\ \overline{\alpha}_{J1} & \overline{\alpha}_{J2} & \ldots & \overline{\alpha}_{JK} \end{bmatrix} \begin{bmatrix} \overline{U}_1 \\ \overline{U}_2 \\ \vdots \\ \overline{U}_K \end{bmatrix} \tag{2.5}$$

If we use the following symbols

$$\mathbf{r} = \begin{bmatrix} \bar{r}_1 \\ \bar{r}_2 \\ \vdots \\ \bar{r}_J \end{bmatrix}; \quad \mathbf{A} = \begin{bmatrix} \overline{\alpha}_{11} & \overline{\alpha}_{12} & \ldots & \overline{\alpha}_{1K} \\ \overline{\alpha}_{21} & \overline{\alpha}_{33} & \ldots & \overline{\alpha}_{2K} \\ \vdots & \vdots & \ddots & \vdots \\ \overline{\alpha}_{J1} & \overline{\alpha}_{J2} & \ldots & \overline{\alpha}_{JK} \end{bmatrix}; \quad \mathbf{U} = \begin{bmatrix} \overline{U}_1 \\ \overline{U}_2 \\ \vdots \\ \overline{U}_K \end{bmatrix}, \tag{2.6}$$

the equation (2.5) will be

$$\mathbf{r} = \mathbf{A} \cdot \mathbf{U}. \tag{2.7}$$

The matrix $\mathbf{A}$is a complex matrix of size $J \times K$ and is called the influence coefficient matrix. The correction weights $\overline{U}_k$ $(k = 1, \ldots, K)$ must be calculated from the balancing condition

$$\bar{r}_j = -\bar{r}_j^A \Rightarrow \bar{r}_j + \bar{r}_j^A = 0. \tag{2.8}$$

In practice there is always residual unbalance vibration $\bar{r}^f$, we have

$$\mathbf{r}^f = \mathbf{r}^A + \mathbf{r}. \tag{2.9}$$

Substituting (2.7) into (2.9), we obtain

$$\mathbf{r}^f = \mathbf{r}^A + \mathbf{A}\,\mathbf{U} \tag{2.10a}$$

or

$$\mathbf{r}_j^f = \mathbf{r}_j^A + \sum_{k=1}^{K} \overline{\alpha}_{jk}\overline{U}_k \quad (j = 1, \ldots, J). \tag{2.10b}$$

If $\mathbf{A}$ is a square and has $\det \mathbf{A} \neq 0$ then from the equation (2.10) we may solve $\mathbf{U}$. In actual balancing, however, the influence coefficient matrix $\mathbf{A}$ is not necessarily square but often a non-square matrix. We will consider the following cases:

a) Case 1: $J = k$ (the number of measured points is equal to the number of the balancing planes). In this case matrix $\mathbf{A}$ is square. Assuming that $\det \mathbf{A} \neq 0$ and from (2.10a) we obtain

$$\mathbf{U} = -\mathbf{A}^{-1}(\mathbf{r}^A - \mathbf{r}^f). \tag{2.11}$$

When $\mathbf{r}^f = \mathbf{0}$, we have the formula to determine the correction weights $\mathbf{U}$

$$\mathbf{U} = -\mathbf{A}^{-1}\mathbf{r}^A. \tag{2.12}$$

According to (2.12) we can determine the correction weights $\mathbf{U}_k$ $(k = 1, \ldots, K)$

b) Cases 2: $J > K$ (the number of measured points is more than the number of the balancing planes). This is the case often met in technical practice provided that $\mathbf{r}^f = \mathbf{0}$, and from (2.10a) we have

$$\mathbf{A}\,\mathbf{U} = -\mathbf{r}^A, \tag{2.13}$$

where $\mathbf{A}$ is non-square. We have J equations and unknown $(K < J)$. The problem has many roots. We have to find out the optimal root. We will adjust the errors and see (2.10a) or (2.10b) as the error equation and use the least square method to deal with a goal that the total sum of squares of errors is minimum.

The total sum of errors is as follows:

$$F = \sum_{j=1}^{J} \left|\mathbf{r}^f\right|^2 = \sum_{j=1}^{J} \overline{r}_j^f \cdot \left(\overline{r}_j^f\right)^*, \tag{2.14}$$

where

$$\begin{aligned} \overline{r}_j^f &= \left(r_j^f\right)' + i\left(r_j^f\right)'', \\ \left(\overline{r}_j^f\right)^* &= \left(r_j^f\right)' - i\left(r_j^f\right)''. \end{aligned} \tag{2.15}$$

Let's mark $\overline{U}_k = U_k' + iU_k''$ then (2.10b) will be:

$$\left.\begin{aligned} \overline{r}_j^f &= \overline{r}_j^A + \sum_{i=1}^{K} \overline{\alpha}_{jk}(U_k' + iU_k'') \\ \left(\overline{r}_j^f\right)^* &= \left(\overline{r}_j^A\right)^* + \sum_{k=1}^{K} \overline{\alpha}_{jk}^*(U_k' - iU_k'') \end{aligned}\right\} \tag{2.16}$$

By substituting (2.16) into (2.14) F is a function with real variables U_k' and U_k'' $(k = 1, \ldots, K)$

$$F = F(U_1' \ldots U_K', U_1'' \ldots U_K''). \tag{2.17}$$

The condition for function F to reach minimum is:

$$\frac{\partial F}{\partial U_k'} = 0; \quad \frac{\partial F}{\partial U_k''} = 0 \quad (k = 1, \ldots, K). \tag{2.18}$$

Thus, as conditions for seeking the correction weights U_k' and U_k'' that minimize equation (2.17) under equations (2.14) and (2.16), the following equations must be obtained:

$$\frac{\partial F}{\partial U_k'} = \sum_{j=1}^{J} \left[\frac{\partial \bar{r}_j^f}{\partial U_k'} (\bar{r}_j^f)^* + \frac{\partial (\bar{r}_j^f)^*}{\partial U_k'} \bar{r}_j^f \right] = 0, \quad (k = 1, \ldots, K), \tag{2.19a}$$

$$\frac{\partial F}{\partial U_k''} = \sum_{j=1}^{J} \left[\frac{\partial \bar{r}_j^f}{\partial U_k''} (\bar{r}_j^f)^* + \frac{\partial (\bar{r}_j^f)^*}{\partial U_k''} \bar{r}_j^f \right] = 0, \quad (k = 1, \ldots, K). \tag{2.19b}$$

By substituting (2.16) into (2.19) and rearranging the results, the following equations are derived:

$$\sum_{j=1}^{J} \left[\overline{\alpha}_{jk} (\bar{r}_j^f)^* + \overline{\alpha}_{jk} \bar{r}_j^f \right] = 2 \sum_{j=1}^{J} \mathrm{Re}(\overline{\alpha}_{jk}^* \bar{r}_j^f) = 0, \ (k = 1, \ldots, K), \tag{2.20}$$

$$\sum_{j=1}^{J} \left[i\overline{\alpha}_{jk} (\bar{r}_j^f)^* - i\overline{\alpha}_{jk}^* \bar{r}_j^f \right] = 2 \sum_{j=1}^{J} \mathrm{Im}(\overline{\alpha}_{jk}^* \bar{r}_j^f) = 0, \ (k = 1, \ldots, K). \tag{2.21}$$

The equations (2.20) may be rewritten as follows

$$\mathrm{Re}(\overline{\alpha}_{1k}^* \bar{r}_1^f + \overline{\alpha}_{2k}^* \bar{r}_2^f + \cdots + \overline{\alpha}_{Jk}^* \bar{r}_J^f) = 0, \ (k = 1, \ldots, K) \tag{2.22}$$

or in the matrix equation as

$$\mathrm{Re} \left\{ \begin{bmatrix} \overline{\alpha}_{11}^* & \overline{\alpha}_{21}^* & \cdots & \overline{\alpha}_{J1}^* \\ \overline{\alpha}_{12}^* & \overline{\alpha}_{22}^* & \cdots & \overline{\alpha}_{J2}^* \\ \vdots & \vdots & \ddots & \vdots \\ \overline{\alpha}_{1K}^* & \overline{\alpha}_{2K}^* & \cdots & \overline{\alpha}_{JK}^* \end{bmatrix} \begin{bmatrix} \bar{r}_1^f \\ \bar{r}_2^f \\ \vdots \\ \bar{r}_J^f \end{bmatrix} \right\} = 0, \tag{2.23}$$

$$\Rightarrow \ \mathrm{Re}\left[(\mathbf{A}^*)^T \mathbf{r}^f \right] = 0. \tag{2.24}$$

With similar changes to those made to equation (2.21) we have

$$\mathrm{Im}\left[(\mathbf{A}^*)^T \mathbf{r}^f \right] = 0, \tag{2.25}$$

where $\left(\mathbf{A}^*\right)^T$ is the transported matrix of the complex combined matrix $\mathbf{A}^*$. Because $\mathbf{A}$ is a matrix of size $J \times K$ then $\left(\mathbf{A}^*\right)^T$ is also of size $K \times J$. The equations (2.24) and (2.25) may be rewritten as follows

$$\left(\mathbf{A}^*\right)^T \mathbf{r}^f = 0. \tag{2.26}$$

By substituting (2.10a) into (2.26), we have

$$\left(\mathbf{A}^*\right)^T \mathbf{r}^A + \left(\mathbf{A}^*\right)^T \mathbf{A}\, \mathbf{U} = 0. \tag{2.27}$$

Noting that $\left(\mathbf{A}^*\right)^T \cdot \mathbf{A}$ is the square matrix of K degree and will not be irregular, therefore from (2.27) we can find the correction weights

$$\mathbf{U} = \left[- \left(\mathbf{A}^*\right)^T \mathbf{A} \right]^{-1} \left(\mathbf{A}^*\right)^T \mathbf{r}^A. \tag{2.28}$$

3. Flow chart of the programme for balancing calculation

The calculation of a system of correction weights is equivalent to the solving of equation (2.28) and shall be implemented with computer software written in C^{++} language. Fig. 2 is a flow chart of the above balancing method. In this method, the influence coefficient can be obtained by either calculation or measurement.

4. Experimental results of verification on models

In order to verify the correctness of the algorithm and the reliability of the computer calculation programme, the tests were made on rotor model KIT, Model 24750 Bently Nevada (USA), equipment LeCroy 9304A QUAD 200 MHz Oscilloscope (USA).

4.1. Experimental model

Rotor KIT is an experimental model for the research of flexible rotor balancing (Fig. 3), including a motor with adjustable speeds between 0 and 10,000 rpm, a shaft, bearings, two balancing disks with caving-off holes which are proportionally located on such disks for mounting the correction weights. Distance between disks and distance between bearings are also adjustable. Vibration at all points on the shaft are measured with non-contact bridge meters.

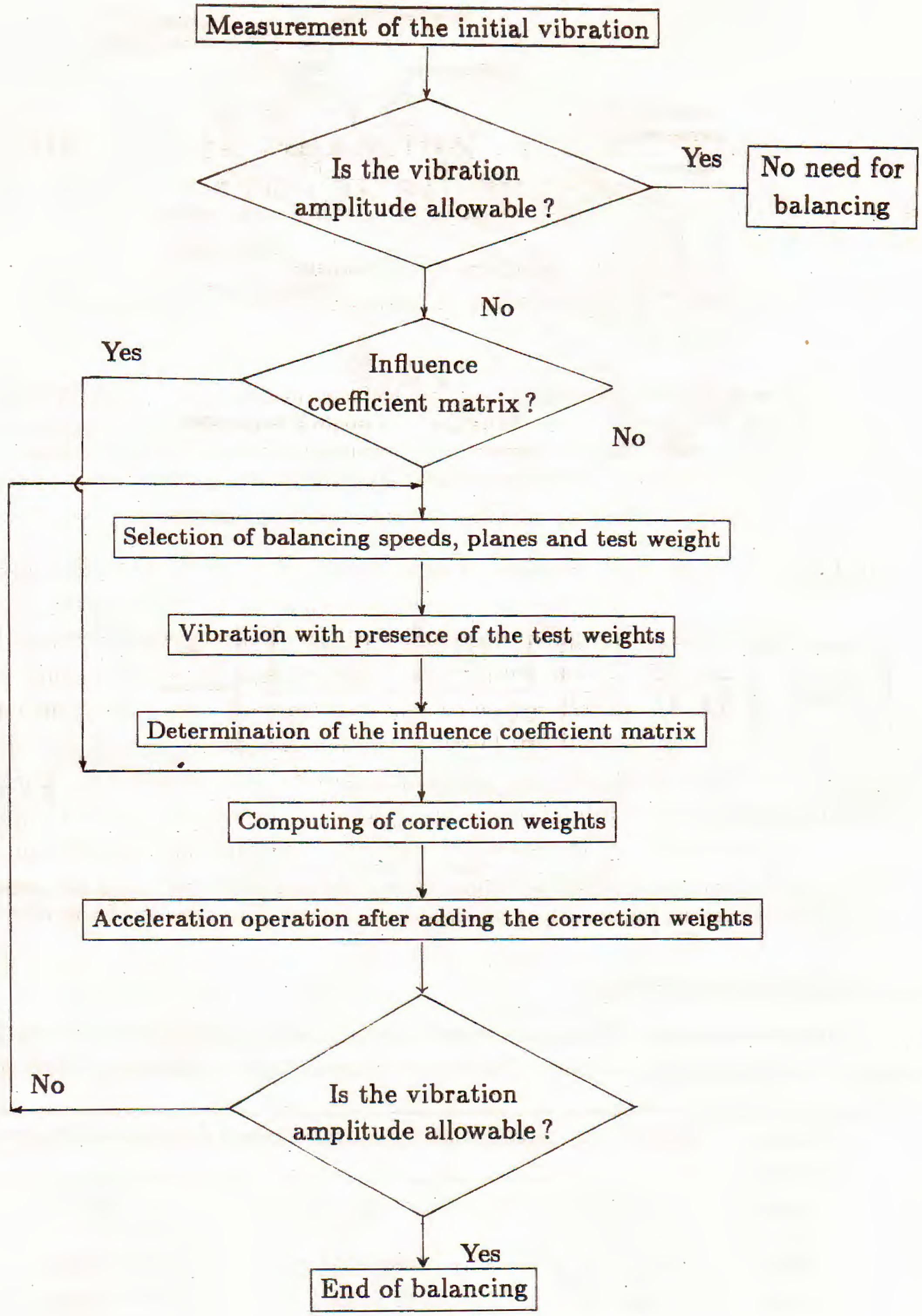

Fig. 2. Flow chart of balancing

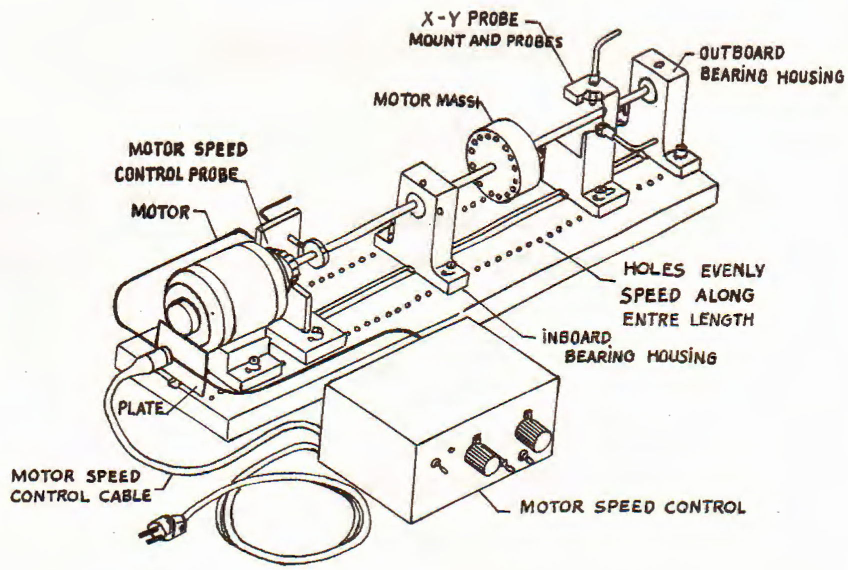

Fig. 3. Model of rotor KIT for the balancing experiment

In Fig. 4 the scheme of the tests is described.

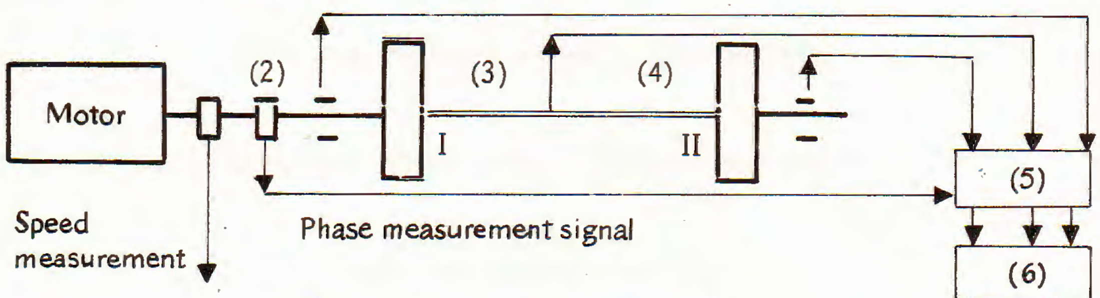

Fig. 4. The principle Scheme of Tests
(0)-Signal for adjustment of the revolution, (1)-key phase, (2), (3), (4) measured points; (I), (II) - balancing planes, (5) - Amplification of signals, (6) - Display of vibration

4.2. Experimentat results

a) Initial vibration. The rotor revolves with certain speeds and vibration is measured at various measured points before balancing as indicated in Tab. 4.1.

Rotor speed, rpm	Vertical amplitudes at measured points $2A/\varphi$, μm/degree		
	(2)	(3)	(4)
3000	85.3/39.3	340/83.8	89.9/172
2700	93/42.6	547.5/64.5	78.13/126.1
2400	242/22.8	878/34.5	119/67.6
1800	46.9/351.5	240/44.7	13.3/264

b) Calculation of balancing added weights. The balancing added weights shall be calculated according to the programe:

$U_1 = 2.16/\underline{18}$ gram/$\underline{\text{degree}}$; $U_2 = 1.17/\underline{274}$ gram/$\underline{\text{degree}}$.

The balancing added weights shall be mounted on the rotor KIT:

$U_1 = 2/\underline{22,5}$ gram/$\underline{\text{degree}}$; $U_2 = 1.2/\underline{270}$ gram/$\underline{\text{degree}}$

Vibration at the measured points at speed of 3000 rpm, after the balancing (Fig. 5a, b, c):

Measured object	Measured point (2)	Measured point (3)	Measured point (4)
$2A/\underline{\varphi}$	$17.6/\underline{198}$	$103.8/\underline{86}$	$55.8/\underline{234}$

The balancing quality [4] for all measured points at speed of $n = 3000$ rpm is $K = 0.71$. The balancing has reached good results and proved the correctness of the algorithm and the programme. The vibrations before balancing and vibrations after balancing are shown in Fig. 5a, b and c.

5. Conclusion

The influence coefficient method allows us to optimize the system of added balancing weights for all balancing planes at various speeds. It does not depend on types of bearing or pivots, does not limit the number of bearings pivots or the number of shafts in one system of shafts, or the modes of eigenvibration of each shaft, each system of shafts. The least squares method was used to deal with errors in the calculation of the correction weights and the determination of the members of the matrix of influence coefficients. We can determine the system of added correction weights to assure the efficiency of the balancing process.

The computer software for calculating the correction weights for the at-the-site balancing of the system of flexible rotors, which has been well verified by tests on various models now allows us to carry out the balancing of the entire system of flexible rotors with high efficiency.

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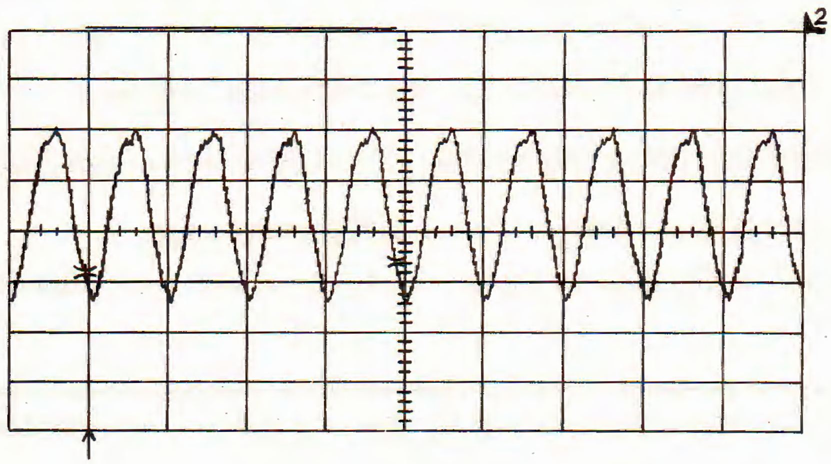
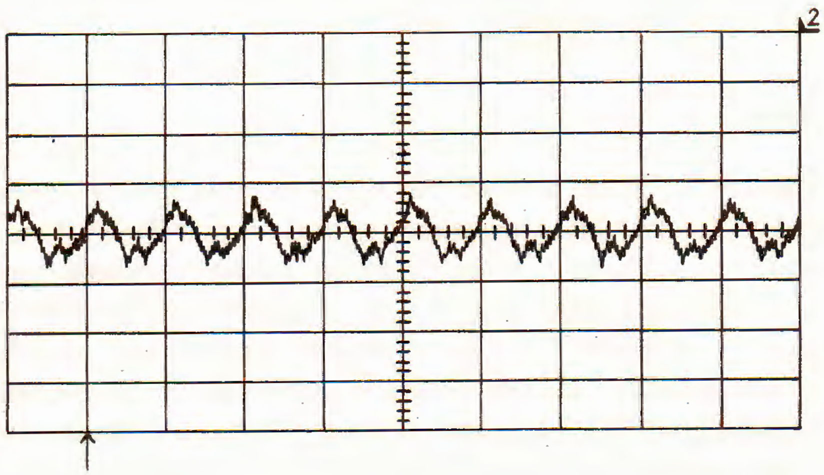


Amplitude at point (2) before balancing at $n = 3000$ rpm

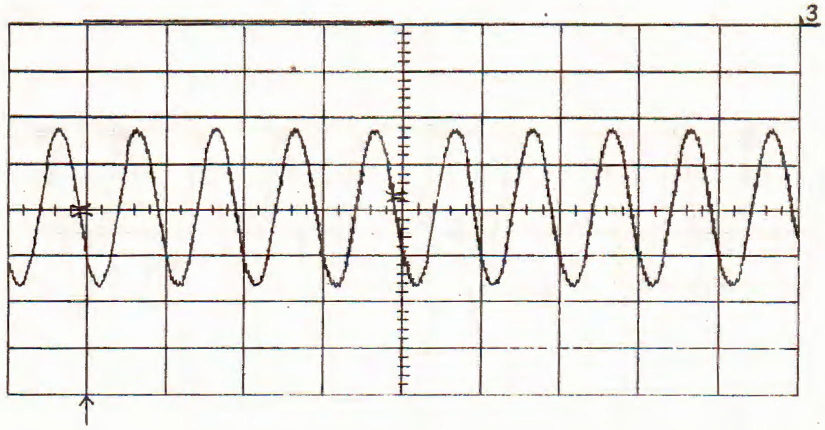


Amplitude at point (2) after balancing at $n = 3000$ rpm

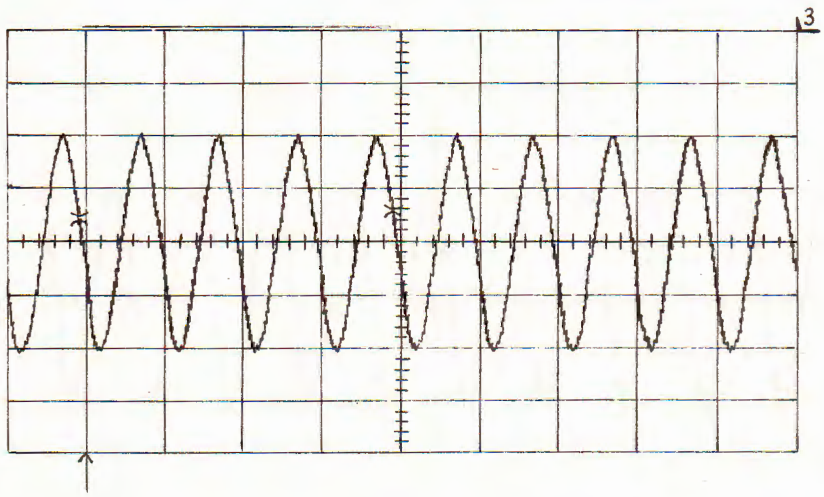
Before balancing: $2A/\underline{\varphi} = 85.8/\underline{39.3}\ \mu m/\underline{\text{degree}}$

After balancing: $2A/\underline{\varphi} = 17.6/\underline{198}\ \mu m/\underline{\text{degree}}$

Fig. 5a. Amplitudes at point (2) before and after balancing



Amplitude at point (3) before balancing at $n = 3000$ rpm

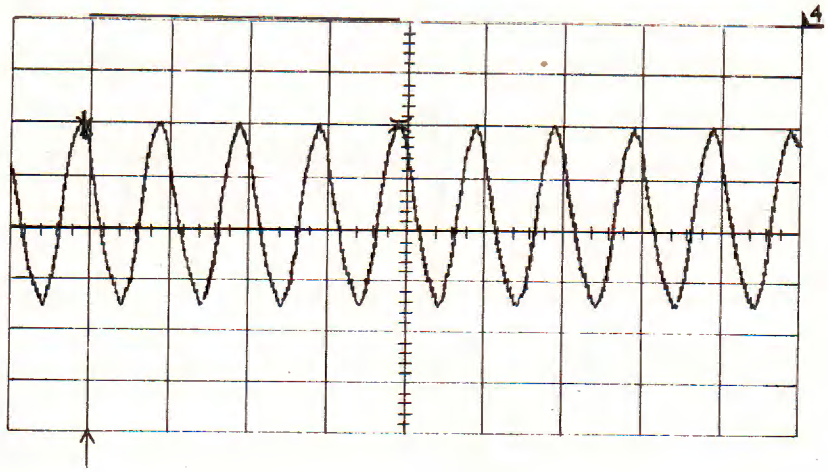


Amplitude at point (3) after balancing at $n = 3000$ rpm

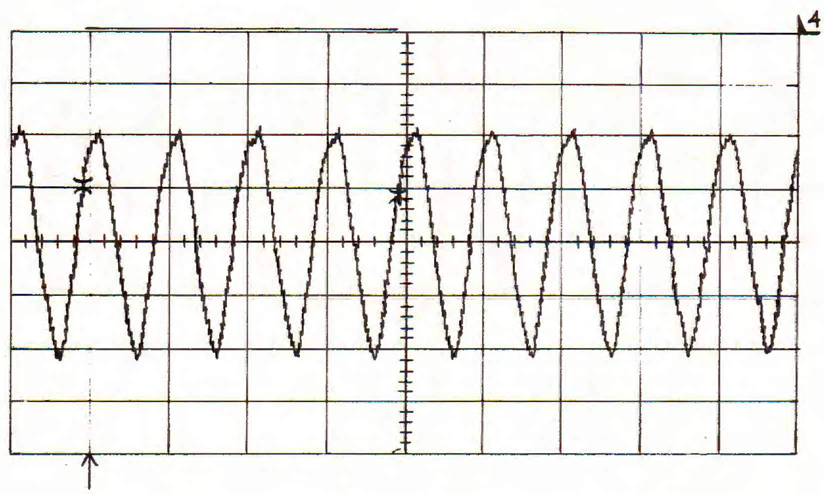
Before balancing: $2A/\underline{\varphi} = 430/\underline{83.8}\ \mu m/\underline{\text{degree}}$

After balancing: $2A/\underline{\varphi} = 103.8/\underline{86}\ \mu m/\underline{\text{degree}}$

Fig. 5b. Amplitudes at point (3) before and after balancing



Amplitude at point (4) before balancing at $n = 3000$ rpm



Amplitude at point (4) after balancing at $n = 3000$ rpm

Before balancing: $2A/\underline{\varphi} = 98.9/\underline{172}\ \mu m/\underline{\text{degree}}$

After balancing: $2A/\underline{\varphi} = 55.8/\underline{234}\ \mu m/\underline{\text{degree}}$

Fig. 5c. Amplitudes at point (4) before and after balancing

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VỀ MỘT PHẦN MỀM TÍNH TOÁN CÂN BẰNG CỦA HỆ RÔTO
ĐÀN HỒI BẰNG PHƯƠNG PHÁP HỆ SỐ ẢNH HƯỞNG

Trong công trình này xét việc sử dụng phương pháp hệ số ảnh hưởng kết hợp với phương pháp bình phương tối thiểu để xác định các vị trí mất cân bằng và tính toán gia trọng cân bằng cho hệ rôto đàn hồi. Một phần mềm tính toán cân bằng tại chỗ cho hệ rôto đàn hồi đã được xây dựng ở Đại học Bách khoa Hà Nội. Từ đó mở ra một khả năng mới cho việc giải quyết các bài toán cân bằng hệ rôto phức tạp ở các nhà máy điện cũng như ở các xí nghiệp có sử dụng các hệ rôto.